Paper Reference(s)

6676/01

Edexcel GCE

Further Pure Mathematics FP3

Advanced Level

Friday 23 June 2006 – Morning
Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP3), the paper reference (6676), your surname, initials and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions on this paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1.

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

Prove by induction, that for all positive integers n,

$$\mathbf{A}^n = \begin{pmatrix} 1 & n & \frac{1}{2}(n^2 + 3n) \\ 0 & 1 & n \\ 0 & 0 & 1 \end{pmatrix}.$$

(5)

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2. (*a*) Find the Taylor expansion of cos $2x$ in ascending powers of $\left(x - \frac{\pi}{4}\right)$ up to and including the term in $\left(x - \frac{\pi}{4}\right)^5$.

(5)

(*b*) Use your answer to (*a*) to obtain an estimate of cos 2, giving your answer to 6 decimal places.

(3)

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3. (*a*) Use de Moivre's theorem to show that

$$\sin 5\theta = \sin \theta (16 \cos^4 \theta - 12 \cos^2 \theta + 1).$$

(5)

(*b*) Hence, or otherwise, solve, for $0 \leq \theta < \pi$,

$$\sin 5\theta + \cos \theta \sin 2\theta = 0.$$

(6)

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4. $$\frac{d^2x}{dt^2} + 3\sin x = 0.$$

At time $t = 0$, $x = 0$ and $\frac{dx}{dt} = 0.4$,

(*a*) Use approximations of the form

$$\left(\frac{dx}{dt}\right)_0 \approx \frac{x_1 - x_0}{h} \text{ and } \left(\frac{d^2x}{dt^2}\right)_0 \approx \frac{x_1 - 2x_0 + x_{-1}}{h^2}, \text{ with } h = 0.1,$$

to obtain estimates of x at $t = 0.1$, $t = 0.2$ and $t = 0.3$.

(5)

(*b*) Find a series solution for x, in ascending powers of t, up to and including the term in t^3.

(4)

(*c*) Use your answer to (*b*) to obtain an estimate of x at $t = 0.3$

(2)

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5. The eigenvalues of the matrix **M**, where

$$\mathbf{M} = \begin{pmatrix} 4 & -2 \\ 1 & 1 \end{pmatrix},$$

are λ_1 and λ_2, where $\lambda_1 < \lambda_2$.

(*a*) Find the value of λ_1 and the value of λ_2.

(3)

(*b*) Find $\mathbf{M}^{-1}$.

(2)

(*c*) Verify that the eignevlaues of $\mathbf{M}^{-1}$ are λ_1^{-1} and λ_2^{-1}.

(3)

A transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is represented by the matrix **M**. There are two lines, passing through the origin, each of which is mapped onto itself under the transformation T.

(*d*) Find cartesian equations for each of these lines.

(4)

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6. The point P represents a complex number z on an Argand diagram, where

$$|z - 6 + 3i| = 3|z + 2 - i|.$$

(*a*) Show that the locus of P is a circle, giving the coordinates of the centre and the radius of this circle.

(7)

The point Q represents a complex number z on an Argand diagram, where

$$\tan [\arg (z + 6)] = \tfrac{1}{2}.$$

(*b*) On the same Argand diagram, sketch the locus of P and the locus of Q.

(5)

(*c*) On your diagram, shade the region which satisfies both

$$|z - 6 + 3i| > 3|z + 2 - i| \text{ and } \tan [\arg (z + 6)] > \tfrac{1}{2}.$$

(2)

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7. The points A, B and C lie on the plane Π_1 and, relative to a fixed origin O, they have position vectors

$$\mathbf{a} = \mathbf{i} + 3\mathbf{j} - \mathbf{k}, \quad \mathbf{b} = 3\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} \quad \text{and} \quad \mathbf{c} = 5\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$$

respectively.

(*a*) Find $(\mathbf{b} - \mathbf{a}) \times (\mathbf{c} - \mathbf{a})$.

(4)

(*b*) Find an equation for Π_1, giving your answer in the form $\mathbf{r.n} = p$.

(2)

The plane Π_2 has cartesian equation $x + z = 3$ and Π_1 and Π_2 intersect in the line l.

(*c*) Find an equation for l, giving your answer in the form $(\mathbf{r} - \mathbf{p}) \times \mathbf{q} = \mathbf{0}$.

(4)

The point P is the point on l that is the nearest to the origin O.

(*d*) Find the coordinates of P.

(4)

TOTAL FOR PAPER: 75 MARKS

END